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Predicting the sound reduction index of finite size specimen by a simplified spatial windowing technique

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ABSTRACT

The transfer matrix technique is an efficient tool for calculating sound transmission through multilayered structures. However, due to the assumption of infinite size layers important discrepancies may be found between predicted and experimental data. The spatial windowing technique introduced by Villot et al. [Predicting the acoustical radiation of finite size multi-layered structures by applying windowing on infinite structures, Journal of Sound and Vibration 245 (2001) 433–455] has shown to give data much closer to measurement results than other measures, such as limiting the maximum angle of incidence when integrating to obtain the sound reduction index for diffuse incidence. Using a two-dimensional spatial window, also including the azimuth angle implies, however, that two double numerical integrations must be performed. As predicted results are compared with laboratory data, where the aspect ratio of the test object is required to be less than 1:2, a simplified procedure may be applied involving two single integrals only. It is shown that the accuracy in the end result may in practice be maintained by this simplified procedure.

0. Introduction

For predicting the diffuse field sound transmission through a single plane homogeneous structure, such as a partition between rooms, a number of formulas may be found in the literature, see e.g. Ref. [1]. Tools for predicting the sound transmission loss, specified by the sound reduction index, of multilayered structures are normally found only for special cases. Describing the transmission through the different layers using a classical wave approach, models using the transfer matrix technique have, however, been shown to be an efficient tool for predicting the performance of general multilayered structures, at least when flexibility and computing time are of concern. However, due to the assumption of infinite size layers important discrepancies may be found between predicted and experimental data. The spatial windowing technique introduced by Villot et al. [2] has been shown to give data much closer to measurement results than other simple procedures, such as, when integrating over the angle of incidence, either using a fixed upper limit, or a frequency and area dependent one. A modification of this technique has later been presented by Villot and Guigou-Carter [3] including one important change, which will be addressed below when presenting the theory.

Another practice has been to include a Gaussian distribution factor in Eq. (6) below, see Refs. [4,5] and works have also been performed combining this procedure with the spatial windowing technique [6,7]. This will, however, not be done here as the purpose of this note is to compare the results given by Villot et al. with results using a simplified windowing

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technique. Double leaf constructions, e.g. double glazings are used as examples when comparing measured and predicted results, also using measured data from two other sources, Rasmussen [8] and Hongisto et al. [9].

Finally, another interesting approach to account for the lateral size effect using the transfer matrix method is presented by Atalla et al. [10], comparing predicted results with measured data as well as predictions using a mixed FEM–BEM approach.

1. Theory

The basic idea behind the windowing technique presented by Villot et al. [2] is to calculate the radiated power where only an area S (of length L_x and width L_y) of an infinite structure contributes to the sound radiation. Assuming a propagating structural wave of wavenumber k_F, the velocity field in the wavenumber domain is defined by taking the spatial Fourier transform, giving

$$\tilde{v}(k_x,k_y) = \hat{v}L_xL_y\frac{\sin((k_x-k_F\cos\psi)L_x/2)}{(k_x-k_F\cos\psi)L_x/2}\cdot\frac{\sin((k_y-k_F\sin\psi)L_y/2)}{(k_y-k_F\sin\psi)L_x/2}, \tag{1}$$

where ψ is the azimuth angle for k_F. This result is used to calculate the radiated power (with reference to Fahy [11]) giving

$$W(k_F,\psi) = \frac{\rho_0c_0}{8\pi^2}\int_0^{k_0}\int_0^{2\pi}\frac{|\tilde{v}(k_x,k_y)|^2}{\sqrt{k_0^2-k_r^2}}k_0k_r\,\mathrm{d}\phi\,\mathrm{d}k_r \quad (k_x = k_r\cos(\phi),\;\; k_y = k_r\sin(\phi)), \tag{2}$$

where k_0, ρ_0 and c_0 are the wavenumber, density and wave speed in the air surrounding the structure. One may then calculate the radiation factor (or radiation efficiency) $\sigma(k_F,\psi)$ by a double integral, which may be written as

$$\sigma(k_F,\psi) = \frac{S}{\pi^2}\int_0^{k_0}\int_0^{2\pi}\frac{\sin^2[(k_r\cos\phi-k_F\cos\psi)L_x]}{[(k_r\cos\phi-k_F\cos\psi)L_x]^2}\cdot\frac{\sin^2[(k_r\sin\phi-k_F\sin\psi)L_y]}{[(k_r\sin\phi-k_F\sin\psi)L_y]^2}\frac{k_0k_r}{\sqrt{k_0^2-k_r^2}}\,\mathrm{d}\phi\,\mathrm{d}k_r. \tag{3}$$

The spatial windowing may now be applied directly to the problem of calculating the sound reduction index of a structure assuming that we know the transmission factor τ_{inf} of the corresponding infinite structure. For an incident acoustic plane wave, the wavenumber k_F is given by $k_F = k_0\sin\theta$ and the radiation factor for the infinite system (no spatial windowing) is given by

$$\sigma_{\mathrm{inf}} = \frac{1}{\cos\theta}. \tag{4}$$

The idea is now to apply the spatial windowing to the radiation process and also to find the effect of spatially windowing the incident pressure field, thereby correcting the transmission factor of the infinite structure to obtain the corresponding transmission factor of the finite structure, thus

$$\tau_{\mathrm{finite}}(\theta,\psi) = \tau_{\mathrm{inf}}[\sigma(k_0\sin\theta,\psi)\cos\theta]^2 \tag{5}$$

and finally calculate the transmission factor for diffuse field excitation by the usual expression

$$\tau_{\mathrm{diffuse}} = \frac{\int_0^{\theta_{\mathrm{lim}}}\int_0^{2\pi}\tau_{\mathrm{finite}}(\theta,\psi)\sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\psi}{\int_0^{\theta_{\mathrm{lim}}}\int_0^{2\pi}\sin\theta\cos\theta\,\mathrm{d}\theta\,\mathrm{d}\psi}, \tag{6}$$

where θ_{lim} is the limiting angle of the diffuse field. This may here be set to 90° as opposed to the common practice, as discussed above, of using a fixed limit of 78°–80°, or to make the upper limit a function of frequency, to get a better fit to experimental data. As shown by Villot et al. and also by others, see e.g. Refs. [6,7], the spatial windowing technique has proved efficient to reduce the gap between predicted and experimental results, not only for single partitions but also for multilayer types, e.g. double glazing.

However, as pointed out in the introduction, an important modification of this procedure is given by Villot and Guigou-Carter [3] substituting Eq. (5) by

$$\tau_{\mathrm{finite}}(\theta,\psi) = \tau_{\mathrm{inf}}\cdot\sigma(k_0\sin\theta,\psi)\cos\theta. \tag{7}$$

As opposed to Eq. (5), which implies spatial windowing of both the exiting sound field and the structure velocity field, only the latter is used. This reduces the effect of the spatial windowing and seems to give a better fit to the measured results, especially at the lower frequencies. When presenting results, we shall distinguish between results using Eqs. (5) and (7) by marking the latter with a small "s" (for single).

As the dimensions L_x and L_y of most practical partitions, certainly the specimens measured in the laboratories, are not too different, what are the implications of using a typical dimension $L = (L_x\cdot L_y)^{1/2}$ to characterize the window, reverting to the one-dimensional case? It is the purpose of this note to show that such a simple prediction model is justified due to easy computation and minor differences as compared with results using the formulas above. It should be borne in mind, however, that this simplification does not apply if orthotropic materials are included.

In the one-dimensional case, the velocity field given in Eq. (1) may be written

$$\tilde{v}(k) = \hat{v}L\frac{\sin\left[(k-k_F)\frac{L}{2}\right]}{(k-k_F)\frac{L}{2}} \tag{8}$$

and instead of Eq. (2) we get

$$W(k_F) = \frac{\rho_0 c_0 k_0}{4\pi}\int_0^{k_0}\frac{|\tilde{v}(k_r)|^2}{\sqrt{k_0^2-k_r^2}}\,\mathrm{d}k_r. \tag{9}$$

The simplified expression for the radiation factor will then be

$$\sigma(k_F) = \frac{Lk_0}{2\pi}\int_0^{k_0}\frac{\sin^2\left[(k_r-k_F)\frac{L}{2}\right]}{\left[(k_r-k_F)\frac{L}{2}\right]^2\cdot\sqrt{k_0^2-k_r^2}}\,\mathrm{d}k_r. \tag{10}$$

Calculating the transmission factor for diffuse incidence in this case will follow the same procedure as used by Villot et al. as shown above, however, also reducing Eq. (6) to a single integral over θ.

2. Measurements and predicted results

The purpose of this note is primarily to reuse the material data and the results given by Villot et al. [2] and compare those with the ones obtained using the simplified expression for the radiation factor, given by Eq. (10). Some experimental results from other sources, [8,9], are also compared with predictions using this simplified procedure.

Fig. 1 shows measured and predicted results of the sound transmission index of a small aluminum plate of dimensions $1.1 \times 1.4\,\mathrm{m}^2$, where measured and predicted results of a spatially windowed system are reproduced from Fig. 12 by Villot et al. [2]. In the same figure predicted results using the simplified approach by setting the dimension $L \approx 1.24\,\mathrm{m}$ are shown. It should be noted that Eq. (5) is used in both predictions. As seen, there is a minute difference between the results from these two methods. The weighted sound reduction index R_w for the frequency range 100–3150 Hz is also calculated. Rounded to the nearest integer, R_w is equal to 24 dB whereas both predictions give 23 dB.

As expected, the accuracy of the prediction methods is less when a multilayered system is addressed. Here we shall present examples on glazings, the common air-filled double ones having different glass thickness. Certainly, using a transfer matrix method, there is no means to account for any structural couplings along the boundary but at low frequencies acoustic coupling across the air gap will be dominant.

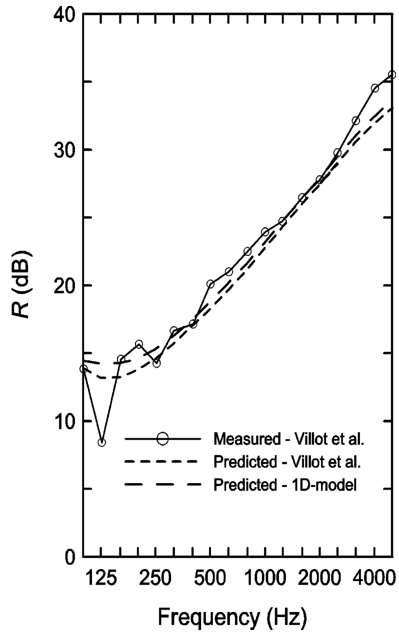

Fig. 1. Sound transmission index of an aluminum plate, measured and predicted results reproduced from Fig. 12 in Villot et al. [2], together with predicted results using a one-dimensional spatial window.

The first example shows the reduction index of a double glazing; see Fig. 2, which is again reproduced from data given in Fig. 14 of the paper by Villot et al. [2]. As stated in their paper, damping in the air gap is included but unfortunately not specified together with other material characteristics. As for the predicted results using the one-dimensional approach, we have here used Eq. (7) instead of Eq. (5) and damping is introduced by a complex propagation coefficient for the air in the gap, provisionally setting the power attenuation coefficient equal to 0.2 m^{-1} independent of frequency. The latter approach is maybe slightly better when comparing with the measurement results but an even better fit may be obtained if one assumes that the effective thickness of the air gap is a little larger than the nominal one, i.e. 14–15 mm instead of 12 mm. The weighted sound reduction index R_w is here 30 dB using the measured data, 29 and 28 dB for the predicted ones, where the latter figure applies to the one-dimensional case.

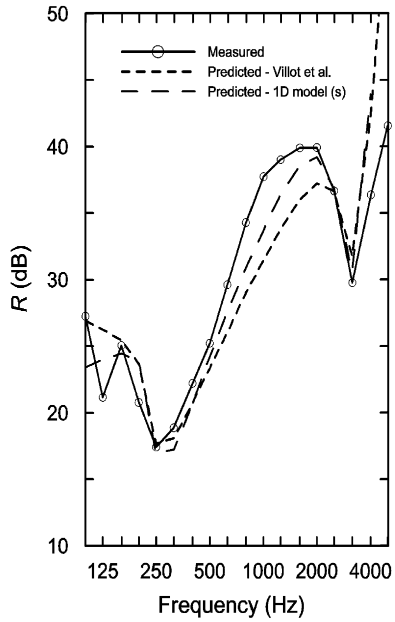

Fig. 2. Sound reduction index of a double glazing (4–12 mm (air)—4 mm), measurement and predicted result reproduced from Fig. 14 in Villot et al. [2], together with predicted result using a one-dimensional spatial window with a single weighting (Eq. (7)).

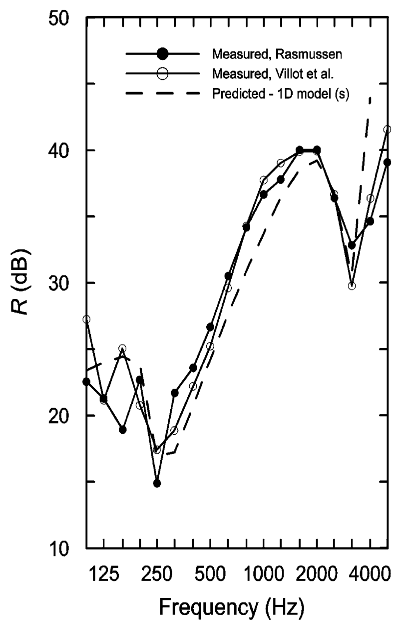

Fig. 3. Sound reduction index of nominally identical double glazing (4–12 mm (air)—4 mm). Measured data from Refs. [2] and [8]. Predicted results using a one-dimensional spatial window with a single weighting (Eq. (7)).

The double glazing used in this example is a quite common one and it could therefore be interesting to compare the measured results with similar measurements from another laboratory. The examples shown in Figs. 3 and 4 are taken from an extensive series of measurements by Rasmussen [8]. In Fig. 3 the measured data and the predicted ones using the one-dimensional spatial window are taken from Fig. 2 and plotted together with measured result from Ref. [8]. As seen, the difference between measured results from these two laboratories, on nominally equal products, is quite small. The dimensions of the test openings were slightly different, the dimensions were $1.2 \times 1.2\,m^2$ in the latter case. The measured data in Fig. 3, taken from Ref. [8], gives an R_w of 31 dB.

Fig. 4 shows measured and predicted results for a double glazing where the thickness of both glass panes is increased to 8 mm. The measured data gives here an R_w of 34 dB whereas the predicted data gives 35 dB.

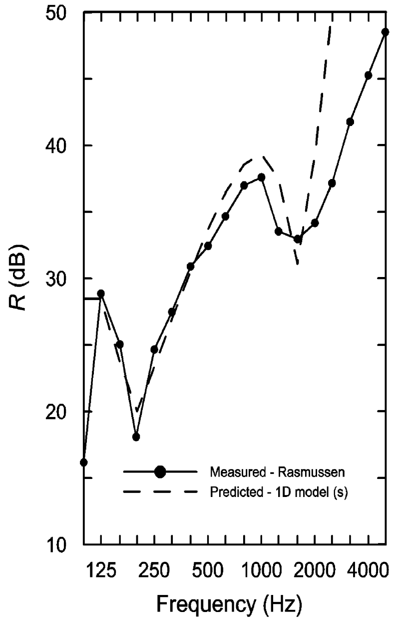

Fig. 4. Sound reduction index of a double glazing (8–12 mm (air)—8 mm), measured [8] and predicted results using a one-dimensional spatial window with a single weighting (Eq. (7)).

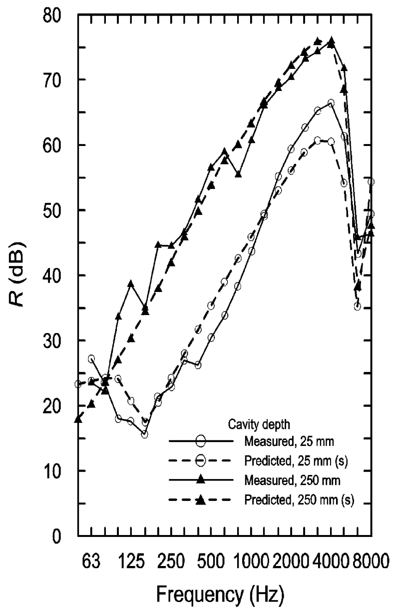

Fig. 5. Sound reduction index of a double leaf construction of 2 mm steel plates with an air cavity of thickness 25 and 250 mm, respectively. Measured results reproduced from Fig. 4a in Ref. [9]. Predicted results using a one-dimensional spatial window with a single weighting (Eq. (7)).

Finally, a couple of results is taken from an extensive series of measurements by Hongisto et al. [9] on an experimental double leaf construction of 2 mm steel plates, mounted in an opening of dimensions 1105×2250 mm, i.e. with an aspect ratio of $\approx 1{:}2$. Fig. 5 shows measured and predicted results for the case of an air filled cavity between the plates of depth 25 and 250 mm, respectively. The measured data gives an R_w of 34 and 56 dB, respectively, whereas the predicted data gives 37 and 52 dB, respectively. The attenuation coefficient used for the air in the cavity is identical to the one used in the cases of double glazing.

3. Conclusions

A simplified windowing technique, based on the work of Villot et al. [2], is used in a transfer matrix model to predict the sound reduction index of multilayered structures in the form of double leaf constructions with an air space. Predicted results compares favorably with results using the full model by Villot et al. and also with measured results on constructions where the aspect ratio of the test specimen extends to 1:2.

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